Simulare Bacalaureat 2025 - Rezolvare M1

Subiectul I

1. $(b_n)_{n\geq 1}$

$$\left.\begin{array}{l} b_3 = 40 \\ b_4 = 80 \end{array}\right| \Rightarrow q = 2 \text{ (rația)}$$

$$b_3 = b_1 \cdot q^2 \Rightarrow 40 = b_1 \cdot 4 \Rightarrow \boxed{b_1 = 10}$$

2. $f: \mathbb{R} \to \mathbb{R}, \ f(x) = x^2 - 2x + m$

$G_f \cap Ox$ în 2 pct. $\Leftrightarrow$ $f(x) = 0$ are 2 sol. reale dist. $\Leftrightarrow$ $\Delta > 0$

$$x^2 - 2x + m = 0$$

$$\Delta = 4 - 4m > 0 \Leftrightarrow 4 > 4m \ | : 4$$

$$1 > m \Rightarrow \boxed{m \in (-\infty, 1)}$$

3. $$3^x + 2 \cdot 3^{x+1} = 63$$

$$\Leftrightarrow 3^x + 6 \cdot 3^x = 63 \Rightarrow 7 \cdot 3^x = 63 \ | : 7$$

$$3^x = 9 \Rightarrow \boxed{x = 2}$$

4. Cazuri posibile: $99 - 10 + 1 = 90$

Cazuri fav. : $10^2 \to 31^2 \Rightarrow 31-10+1=22$
$(100) \qquad (961)$

$$\Rightarrow P = \frac{\text{cazuri fav.}}{\text{cazuri pos.}} = \frac{22}{90} = \frac{11}{45}.$$

5 $A(1,2)$, $B(7,4)$ | $\overrightarrow{AB} = 2\overrightarrow{AC}$

$\Leftrightarrow (x_B - x_A)\cdot\vec{i} + (y_B - y_A)\cdot\vec{j} =$
$= 2\left((x_C - x_A)\cdot\vec{i} + (y_C - y_A)\cdot\vec{j}\right)$

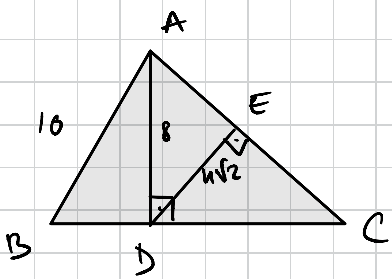
$\Rightarrow 6\vec{i} + 2\vec{j} = 2(x_C - 1)\cdot\vec{i} + 2(y_C - 2)\cdot\vec{j}$

$\Rightarrow \begin{cases} x_C - 1 = 3 \\ y_C - 2 = 1 \end{cases} \Rightarrow C(4,3)$

$\overrightarrow{OD} = \overrightarrow{CB} \Leftrightarrow x_D\cdot\vec{i} + y_D\cdot\vec{j} = 3\cdot\vec{i} + \vec{j}$

$\Rightarrow \boxed{D(3,1)}$

6



În ΔADC, $m(\measuredangle D) = \frac{\pi}{2}$
DE – înălțime

$\Rightarrow DE = \frac{AC}{2} \Rightarrow \boxed{AC = 8\sqrt{2}}$

Avem în ΔADB, $m(\measuredangle D) = \frac{\pi}{2}$

Pitagora $\Rightarrow$ $AD^2 + BD^2 = AB^2 \Rightarrow \boxed{BD = 6}$

Analog în $\Delta ADC \overset{Pit.}{\Rightarrow} AD^2 + DC^2 = AC^2$

$$64 + DC^2 = 128$$

$$\Rightarrow \boxed{DC = 8}$$

$BC = 6 + 8 = 14$

$$\mathcal{A}_{\Delta ABC} = \frac{b \cdot h}{2} = \frac{14 \cdot 8}{2} = 7 \cdot 8 = 56.$$

Subiectul II

① $A(a) = \begin{pmatrix} a & 1 & -a \\ 3 & 1 & -2 \\ 1 & -3 & a \end{pmatrix}$

a) $A(0) = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 1 & -2 \\ 1 & -3 & 0 \end{pmatrix}$

$\Rightarrow \det(A(0)) = 0 + 0 - 2 - 0 - 0 - 0 = -2$

(Reg. Δ)

b) Sist. are sol. unică $\Leftrightarrow \det(A(a)) \neq 0$

$\det(A(a)) = a^2 + 9a - 2 + a - 3a - 6a$

$= a^2 + a - 2 = 0$

$\Leftrightarrow \Delta = 1 + 8 = 9 \Rightarrow a_{1,2} = \frac{-1 \pm 3}{2}$

$\Rightarrow a_1 = 1, \quad a_2 = -2$

Deci, sist. are sol. unică $\Leftrightarrow$ $\boxed{a \in \mathbb{R} \setminus \{-2, 1\}}$

c) $a = 1 \Rightarrow \det(A(1)) = 0$

Avem minorul principal $m_p = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1 - 3 = -2 \neq 0$

$\Rightarrow m_{carac.} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & -3 & -3 \end{vmatrix} = -3 - 9 + 1 - 1 + 9 + 3 = 0$

$\Rightarrow$ sist. este compatibil nedet.

ec. princ. (1), (2)

nec. princ. : x, y

notăm $z = \alpha \in \mathbb{R}$

$$\Rightarrow \begin{cases} x + y - \alpha = 1 \\ 3x + y - 2\alpha = 1 \end{cases} \Leftrightarrow \begin{cases} x + y = 1 + \alpha \\ 3x + y = 1 + 2\alpha \end{cases} \ominus$$

$2x = \alpha$

$\Rightarrow x = \frac{\alpha}{2}$

$\frac{\alpha}{2} + y = 1 + \alpha \Rightarrow y = 1 + \frac{\alpha}{2}$

$\Rightarrow$ sol. sist. este $\left(\frac{\alpha}{2}, 1 + \frac{\alpha}{2}, \alpha\right)$, $\alpha \in \mathbb{R}$

Fie $(x_1, y_1, z_1) = \left(\frac{\alpha_1}{2}, 1 + \frac{\alpha_1}{2}, \alpha_1\right)$

$$(x_2, y_2, z_2) = \left(\frac{\alpha_2}{2}, 1+\frac{\alpha_2}{2}, \alpha_2\right)$$

$$\Rightarrow \begin{cases} 1 + \frac{\alpha_1}{2} = \frac{\alpha_2}{2} \quad | \cdot 2 \\ \alpha_1 = 1 + \frac{\alpha_2}{2} \quad | \cdot 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2 + \alpha_1 = \alpha_2 \\ 2\alpha_1 = 2 + \alpha_2 \end{cases} \quad \ominus$$

$$\alpha_1 - 2 = 2 \Rightarrow \alpha_1 = 4 \Rightarrow \alpha_2 = 6$$

$\Rightarrow$ Sol. cerute sunt : $(x_1, y_1, z_1) = (2, 3, 4)$

$(x_2, y_2, z_2) = (3, 4, 6)$

② $M = (0, +\infty)$

$$x * y = \sqrt{xy} + \frac{1}{\sqrt{xy}} + \frac{x+y}{2} - 2$$

a) $1 * 4 = \sqrt{4} + \frac{1}{\sqrt{4}} + \frac{5}{2} - 2 = 2 + \frac{6}{2} - 2 = 3.$

b) $x * x = 1$

$$x * x = \sqrt{x^2} + \frac{1}{\sqrt{x^2}} + \frac{2x}{2} - 2$$

$x \in (0, +\infty)$!

$$\Rightarrow x * x = x + \frac{1}{x} + x - 2 = 2x + \frac{1}{x} - 2 = 1$$

$\Leftrightarrow \quad 2x + \frac{1}{x} = 3 \quad | \cdot x$

$$2x^2 - 3x + 1 = 0$$

$$\Delta = 9 - 8 = 1 \Rightarrow x_{1,2} = \frac{3 \pm 1}{4}$$

$\Rightarrow \quad x_1 = 1 \quad , \quad x_2 = \frac{1}{2}.$

c) $N = [1, +\infty)$ p.s. a lui M în rap. cu „$*$"

$\Leftrightarrow \quad \forall \; x, y \in N, \quad x * y \in N$

Fie $x, y \in N$

$\Rightarrow \begin{cases} x \geq 1 \\ y \geq 1 \end{cases} \odot$

$xy \geq 1 \quad | \sqrt{}$

$\sqrt{xy} \geq 1$

Avem $t + \frac{1}{t} \geq 2 \quad \forall t > 0$

$\Rightarrow \sqrt{xy} + \frac{1}{\sqrt{xy}} \geq 2 \quad \forall x, y \in [1, +\infty)$

Totodată, $x + y \geq 2 \quad | : 2 \quad \Rightarrow \quad \frac{x+y}{2} \geq 1$

$\Rightarrow$ Adunând, $\sqrt{xy} + \frac{1}{\sqrt{xy}} + \frac{x+y}{2} \geq 3 \quad | -2$

$\Rightarrow \quad x * y \geq 1 \Rightarrow x * y \in N$

$\Rightarrow$ N este p.s. a lui M în rap. cu „$*$".

Subiectul III

(1) $f: (0,+\infty) \to \mathbb{R},\ f(x) = \frac{2x-2}{x+2} + \ln \frac{x+2}{x}$

a) $f'(x) = \frac{(2x-2)'(x+2) - (2x-2)(x+2)'}{(x+2)^2} + \frac{x}{x+2} \cdot \left(\frac{x+2}{x}\right)'$

$= \frac{2(x+2) - 2x + 2}{(x+2)^2} + \frac{x}{x+2} \cdot \frac{x - x - 2}{x^2}$

$= \frac{6}{(x+2)^2} - \frac{2}{x(x+2)} = \frac{6x - 2(x+2)}{x(x+2)^2} = \frac{4x-4}{x(x+2)^2}$

$= \frac{4(x-1)}{x(x+2)^2} \quad \forall\ x \in (0,+\infty)$

b) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{2x-2}{x+2} + \ln \frac{x+2}{x}$

$\overset{\frac{\infty}{\infty}}{\underset{l'H}{=}} \lim\limits_{x\to\infty} \frac{2}{1} + \ln \frac{x+2}{x} = 2 + \ln\left[\lim\limits_{x\to\infty} \frac{x+2}{x}\right]$

$= 2 + \ln 1 = 2$

$\Rightarrow y = 2$ asimp. oriz. la $+\infty$

c) Fie $g: (0,+\infty) \to \mathbb{R},\ g(x) = f(x) - m$

$g'(x) = f'(x) = 0 \iff \frac{4(x-1)}{x(x+2)^2} = 0 \iff x = 1$

Avem

x	0		1		$+\infty$
$g'(x)$		$-\ -\ -\ -\ -$	0	$+\ +\ +\ +\ +\ +$	
$g(x)$		$+\infty \searrow$	$\ln 3 - m$	$\nearrow$	$2-m$
Rolle		$+$	$+$		$+$

Avem $g(1) = \dfrac{2\cdot 1 - 2}{1+2} + \ln \dfrac{3}{1} - m$

$$= \ln 3 - m$$

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} f(x) - m = 2 - m$$

$$\lim_{\substack{x\to 0\\ x>0}} g(x) = \lim_{\substack{x\to 0\\ x>0}} \frac{2x-2}{x+2} + \ln\frac{x+2}{x} - m$$

$$= +\infty$$

Avem din tabel și g - continuă:

g - descrescătoare pe $(0, 1]$
- crescătoare pe $[1, +\infty)$

$\Rightarrow$ Din șirul lui Rolle, pentru ca ec. $g(x) = 0$ să <u>nu</u> aibă soluții, avem același semn în $x = 1$ și $x \to \infty$. Deci, $\ln 3 - m > 0$ și $2 - m > 0$

$$\Rightarrow \begin{cases} \ln 3 > n \\ 2 > n \end{cases} \Bigg| \text{ cum } n \in \mathbb{N}$$

$$\Rightarrow \underline{n \in \{0, 1\}}$$

② $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \dfrac{x^2}{2x^2+1}$

a) $$\int_{-1}^{2} (2x^2+1)\, f(x)\, dx = \int_{-1}^{2} x^2\, dx = \frac{x^3}{3}\Big|_{-1}^{2}$$

$$= \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3.$$

b) $$I = \int_0^2 \sqrt{f(x)}\, dx = \int_0^2 \sqrt{\frac{x^2}{2x^2+1}}\, dx$$

$$= \frac{1}{4} \int_0^2 \frac{4x}{\sqrt{2x^2+1}}\, dx$$

Not. $t = 2x^2+1$ $\begin{cases} x=0 \Rightarrow t=1 \\ x=2 \Rightarrow t=9 \end{cases}$

$dt = 4x\, dx$

$$\Rightarrow I = \frac{1}{4} \int_1^9 \frac{dt}{\sqrt{t}} = \frac{1}{4} \int_1^9 t^{-\frac{1}{2}}\, dt$$

$$= \frac{1}{4} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \Bigg|_1^9$$

$$= \frac{\sqrt{t}}{2} \Bigg|_1^9 = \frac{3}{2} - \frac{1}{2} = 1.$$

c) $$I_n = \int_0^1 \frac{x^n}{f(\sqrt{e^x})}\, dx \quad \forall\ n \in \mathbb{N}^*$$

$$f(\sqrt{e^x}) = \frac{2e^x+1}{e^x} \Rightarrow I_n = \int_0^1 x^n \cdot \frac{2e^x+1}{e^x}\, dx$$

Avem $$\int \frac{2e^x+1}{e^x}\, dx = \int 2\, dx + \int e^{-x}\, dx$$

$$= 2x - e^{-x} + \mathcal{C}$$

$$I_{n+1} = \int_0^1 x^{n+1} \cdot (2x - e^{-x})'\, dx$$

$$= x^{n+1} \cdot \left(2x - \frac{1}{e^x}\right)\Big|_0^1 - \int_0^1 (n+1) \cdot x^n \left(2x - e^{-x}\right) dx$$

$$= 2 - \frac{1}{e} - (n+1) \cdot \int_0^1 x^n \cdot 2x - x^n \cdot e^{-x}\, dx$$

$$= 2 - \frac{1}{e} - (n+1) \cdot 2 \cdot \frac{x^{n+2}}{n+2}\Big|_0^1 + (n+1) \cdot \int_0^1 x^n \cdot e^{-x}\, dx$$

$$= 2 - \frac{1}{e} - \frac{2(n+1)}{n+2} + (n+1) \cdot \int_0^1 x^n \cdot e^{-x}\, dx$$

$$I_n = \int_0^1 x^n \cdot \left(\frac{2e^x+1}{e^x}\right) dx = \int_0^1 x^n (2 + e^{-x})\, dx$$

$$= 2\int_0^1 x^n\, dx + \int_0^1 x^n e^{-x}\, dx$$

Avem $(n+1)\mathring{I}_n - \mathring{I}_{n+1}$

$$= 2(n+1)\cdot\int_0^1 x^n dx + (n+1)\int_0^1 x^n\cdot e^{-x} dx - 2 + \frac{1}{e} + \frac{2(n+1)}{(n+2)}$$

$$- (n+1)\cdot\int_0^1 x^n e^{-x} dx$$

$$= 2(n+1)\cdot\frac{x^{n+1}}{n+1}\Big|_0^1 - 2 + \frac{1}{e} + \frac{2(n+1)}{n+2}$$

$$= \frac{2(n+1)}{n+2} + \frac{1}{e} \quad (\text{q.e.d.})$$