Examenul național de bacalaureat 2025

Matematică M_tehnologic

Simulare

Subiectul I

① $\# a_3 = ?$

$a_1 = 3$, $a_2 = 10$

$a_2 = a_1 + r \Rightarrow r = a_2 - a_1 = 7$

$a_3 = a_1 + 2r = 3 + 2 \cdot 7 = 3 + 14 = 17$

② $f, g : \mathbb{R} \to \mathbb{R}$

$f(x) = 3x - 4$, $g(x) = x + 2$

$\# a = ?$ $f(a) = a + g(2)$

$f(a) = a + g(2) \Leftrightarrow 3a - 4 = a + 2 + 2$

$\Leftrightarrow 3a = a + 8$

$\Leftrightarrow a = 4$

③ $\log_3(10x - 1) = 2$

C.E $10x - 1 > 0 \Rightarrow x > 1/10$

$\log_3(10x - 1) = \log_3 9$

log este o funcție injectivă

$\Rightarrow 10x - 1 = 9$

$\Rightarrow x = 1$

$1 > 1/10 \Rightarrow x = 1$ soluție unică

④ $p - \frac{45}{100}p = 110 \Leftrightarrow \frac{(100-45)p}{100} = 110$

$\Leftrightarrow \frac{55p}{100} = 110 \Leftrightarrow \frac{p}{100} = 2 \Rightarrow p = 2\cdot 100 = 200$ le

⑤ $A(0,4)$, $B(0,-1)$, $C(8,3)$

$AB = \sqrt{0^2 + (-1-4)^2} = 5$

M – mij $BC \Rightarrow M(4,1)$

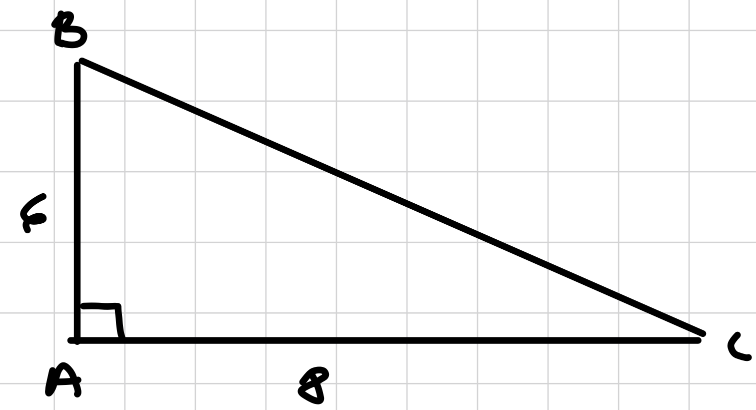
$AM = \sqrt{4^2 + (1-4)^2} = \sqrt{25} = 5$

$\Rightarrow AB = AM$

⑥ ΔABC dreptunghic în A

$AB = 6$, $AC = 8$

$\sin C = \frac{AB}{BC}$



$BC = \sqrt{AB^2 + AC^2}$ (teorema lui Pitagora)

$\Rightarrow BC = \sqrt{36+64} = 10$

$\Rightarrow \sin C = \frac{6}{10} = \frac{3}{5}$

Subiectul al II-lea

1. $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ și $A(a) = \begin{pmatrix} a & 3a \\ a & 2a+3 \end{pmatrix}$, $a \in \mathbb{R}$

a) $\det(A(2)) = \det\left(\begin{pmatrix} 2 & 6 \\ 2 & 7 \end{pmatrix}\right) = 2 \cdot 7 - 2 \cdot 6 = 2$

b) $\# x = ?$

$A(1)\ A(1) + 2I_2 = x\ A(1)$

$A(1)\ A(1) + 2I_2 = \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1+3 & 3+15 \\ 1+5 & 3+25 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 18 \\ 6 & 30 \end{pmatrix}$

$= 6 \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix} = 6\ A(1)$

$\Rightarrow x\ A(1) = 6 \cdot A(1) \xRightarrow{\text{injectivitate}} x = 6$

c) $\# X \in M_2(\mathbb{R})$ a.î. $A(2)\ X\ A(2) = A(0)$

$A(2) = \begin{pmatrix} 2 & 6 \\ 2 & 7 \end{pmatrix}$ și $\det(A(2)) = 2 \neq 0 \Rightarrow A(2)$ inversabilă

$A^{-1}(2)\ \big|\ A(2) \cdot X\ A(2) = A(0)\ \big| \cdot A^{-1}(2)$

$$\Rightarrow X = A^{-1}(2) \cdot A(0) \; A^{-1}(2)$$

$$A^{-1}(2) = \frac{1}{\det(A(2))} A^{*}(2)$$

$$A^{T}(2) = \begin{pmatrix} 2 & 2 \\ 6 & 7 \end{pmatrix} \Rightarrow A^{*}(2) = \begin{pmatrix} +7 & -6 \\ -2 & +2 \end{pmatrix}$$

$$\Rightarrow A^{-1}(2) = \frac{1}{2}\begin{pmatrix} 7 & -6 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 7/2 & -3 \\ -1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 7/2 & -3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 7/2 & -3 \\ -1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & -3\cdot 3 \\ 0 & 1\cdot 3 \end{pmatrix}\begin{pmatrix} 7/2 & -3 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -9 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 7/2 & -3 \\ -1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} (-9)(-1) & (-9)1 \\ 3(-1) & 3\cdot 1 \end{pmatrix} = \begin{pmatrix} 9 & -9 \\ -3 & 3 \end{pmatrix}$$

② $x * y = xy - 2x - 2y + 6$, $x*y$ asociativă

a) $0 * 2 = 0\cdot 2 - 2\, 0 - 2\, 2 + 6 = 0 - 0 - 4 + 6 = -4 + 6 = 2$

b). $\exists\, x = ?\; x \in \mathbb{R}$ a.î. $x * (2x) = 6$

$$x * (2x) = x \cdot 2x - 2\cdot x - 2\cdot 2x + 6$$

$$= 2x^2 - 6x + 6$$

$x * (2x) = 6 \Leftrightarrow 2x^2 - 6x + 6 = 6$

$\Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow x^2 - 3x = 0$

$\Leftrightarrow x(x-3) = 0$

$\Rightarrow x \in \{0, 3\}$

c) e = elem neutru al $*$, $e = 3$

fie x' simetricul lui x in raport cu $*$.

$x' = 4 \Rightarrow (4 * x) = (x * 4) = 3$

$4 * x = 4x - 2 \cdot 4 - 2 \cdot x + 6$

$= 2x - 2$

$4 * x = 3 \Rightarrow 2x - 2 = 3 \Rightarrow x = 5/2$

verificare. $(5/2 * 4) = 4 \cdot 5/2 - 2 \cdot 5/2 - 2 \cdot 4 + 6$

$= 10 - 5 - 8 + 6$

$= 16 - 13 = 3$ D

$\Rightarrow x = 5/2$

Subiectul al III-lea

① fie $f\ (0,+\infty)\to\mathbb{R},\ f(x)=2x^2-2-\ln x$

a). f derivabilă și $f'(x)=(2x^2)'-(2)'-(\ln x)'\ \forall x\in(0,+\infty)$

$$f'(x)=4x-0-\frac{1}{x}=\frac{4x^2-1}{x}\quad \forall x\in(0,+\infty)$$

$$f'(x)=\frac{(2x-1)(2x+1)}{x}\quad \forall x\in(0,+\infty)$$

b). $$\lim_{x\to1}\frac{f(x)+\ln x}{3x-3}=\lim_{x\to1}\frac{2x^2-2}{3x-3}=\lim_{x\to1}\frac{2(x-1)(x+1)}{3(x-1)}$$

$$=\lim_{x\to1}\frac{2\cancel{(x-1)}(x+1)}{3\cancel{(x-1)}}=\lim_{x\to1}\frac{2(x+1)}{3}=\frac{2\ 2}{3}=\frac{4}{3}$$

c) $\#\ \frac{4x^2-1}{2}\geq\ln(2x)\ \forall x\in(0,+\infty)$

$$\frac{4x^2-1}{2}\geq\ln(2x)\iff 4x^2-1\geq2\ln(2x)\ \forall x\in(0,+\infty)$$

$$\iff(2x)^2-1\geq2\ln(2x)\quad\forall x\in(0,+\infty)$$

$$x\in(0,+\infty)\Rightarrow2x\in(0,+\infty)$$

$$\Rightarrow(2x)^2-1\geq2\ln(2x)\ \forall x\in(0,+\infty)\iff$$

$$x^2-1\geq2\ln x\iff x^2-2\ln x-1\geq0\ \forall x\in(0,\infty)$$

fie $g\ (0,+\infty)\to\mathbb{R},\ g(x)=x^2-2\ln x-1$

g derivabilă și $g'(x)=2x-\frac{2}{x}$

$$g'(x) = 0 \iff 2x - \frac{2}{x} = 0 \iff x - \frac{1}{x} = 0$$

$$\iff \frac{x^2 - 1}{x} = 0 \iff x^2 - 1 = 0 \iff x \in \{\pm 1\}$$

$$x \in (0, +\infty) \Rightarrow x = 1$$

$$x^2 - 1 > 0 \quad \forall x \in (-\infty, -1) \cup (1, +\infty)$$

$$x^2 - 1 < 0 \quad \forall x \in (-1, 1)$$

$$x > 0 \quad \forall x \in (0, +\infty)$$

$$\Rightarrow \begin{cases} g'(x) < 0 \quad \forall x \in (0,1) \Rightarrow g \text{ str } \searrow \text{ pe } (0,1) \\ g'(x) = 0 \quad x = 1 \\ g'(x) > 0 \quad \forall x \in (1, +\infty) \Rightarrow g \text{ str } \nearrow \text{ pe } (0,1) \end{cases}$$

$\Rightarrow x = 1$ este punct de minim global

$$\left.\begin{array}{l} \Rightarrow g(x) \geq g(1) \quad \forall x \in (0, +\infty) \\ g(1) = 1 - 2\ln 1 - 1 = 0 \end{array}\right| \Rightarrow g(x) \geq 0 \quad \forall x \in (0, +\infty)$$

$$\Rightarrow x^2 - 2\ln x - 1 \geq 0 \quad \forall x \in (0, +\infty)$$

$$\Rightarrow (2x)^2 - 2\ln(2x) - 1 \geq 0 \quad \forall x \in (0, +\infty)$$

$$\Rightarrow 4x^2 - 1 \geq 2\ln(2x) \quad \forall x \in (0, +\infty)$$

$$\Rightarrow \frac{4x^2 - 1}{2} \geq \ln(2x) \quad \forall x \in (0, +\infty) \quad D$$

② fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^x + 2x + 2$

a) $\int_0^1 (f(x) - 2x)\, dx = \int_0^1 (e^x + 2)\, dx = e^x \Big|_0^1 + 2x \Big|_0^1$

$= e - 1 + 2 - 0 = e + 1$

b) $\int_0^3 \frac{1}{f(x) - e^x}\, dx = \int_0^3 \frac{1}{2x+2}\, dx = \frac{1}{2} \int_0^3 \frac{1}{x+1}\, dx$

$= \frac{1}{2} \cdot \ln(x+1) \Big|_0^3 = \frac{1}{2} (\ln 4 - \ln 1)$

$= \frac{1}{2} \cdot 2 \ln 2 = \ln 2$

c) $\nexists\, a \in \mathbb{R}$ a.î. $\int_0^1 \frac{f(x)}{e^x}\, dx = 5 + \frac{a}{e}$

$\int_0^1 \frac{f(x)}{e^x}\, dx = \int_0^1 \frac{e^x + 2x + 2}{e^x}\, dx = \int_0^1 \left(1 + \frac{2x}{e^x} + \frac{2}{e^x}\right) dx$

$= \underbrace{\int_0^1 1\, dx}_{\alpha} + \underbrace{2\int_0^1 \frac{x}{e^x}\, dx}_{\beta} + \underbrace{2\int_0^1 \frac{1}{e^x}\, dx}_{\Gamma}$

$\alpha = x \Big|_0^1 = 1 - 0 = 1$

$\Gamma = 2 \cdot \int_0^1 \frac{1}{e^x}\, dx = 2 \cdot \int_0^1 e^{-x}\, dx = -2e^{-x} \Big|_0^1 = -\frac{2}{e} + 2$

$$\beta = 2\int_0^1 \frac{x}{e^x}\,dx = -2\int_0^1 x\cdot(e^{-x})'\,dx$$

$$= -2\left(x\cdot e^{-x}\Big|_0^1 - \int_0^1 e^{-x}\,dx \right)$$

$$= -2\left(\frac{1}{e} + e^{-x}\Big|_0^1 \right) = -2\left(\frac{1}{e} + \frac{1}{e} - 1 \right) = -\frac{4}{e} + 2$$

$$\Rightarrow \int_0^1 \frac{f(x)}{e^x}\,dx = 1 - \frac{2}{e} + 2 - \frac{4}{e} + 2 = 5 - \frac{6}{e} = 5 + \frac{a}{e}$$

$$\Rightarrow a = -6 \quad \square$$